## Semileptonic and non-leptonic $B_c$ decays

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#### Abstract

We make predictions for the exclusive semileptonic and the non-leptonic decay widths of the  $B_c$  meson. We evaluate the  $B_c$  semileptonic form factors for different decay channels in a relativistic model, and use factorization to obtain the non-leptonic decay widths.

The recent discovery of the  $B_c$  meson by the CDF Collaboration [1] attracted a great deal of attention. The  $B_c$  meson is very interesting because it carries non-vanishing flavor quantum numbers, and lies below the threshold of the BD decay. Therefore, it can only decay through weak interactions which makes this doubly heavy meson useful for studying the weak decays of heavy flavors. The  $B_c$  production mechanisms, spectroscopy, and decays have been analyzed using different approaches ( see Ref. [2] for a review ).

In a previous paper [3] we have used a relativistic model [4] based on the Bethe-Salpeter Equation (BSE) to evaluate the spectrum of the  $B_c$  meson. No free parameters were used to fit the  $B_c$  spectrum. Instead, all the model parameters had been fixed in previous investigations of other meson spectra. We also evaluated the decay constant of the  $B_c$  meson, the inclusive decay widths of the c-quark and the  $\bar{b}$ -quark together with the annihilation width. Our results agree very well with the CDF results of the  $B_c$  mass and lifetime. We have presented these results with two covariant reductions of the BSE and observed little dependence on the choice of the reduction especially in the heavy flavor sector.

In this paper we evaluate the exclusive semileptonic  $B_c \to P(V)e\nu$  and two-body non-leptonic  $B_c \to PP$ , PV, VV decay widths, where P (V) denotes a pseudoscalar (vector) meson. We use our model to calculate the semileptonic form factors for different decay channels. We then use factorization to obtain the non-leptonic decay widths. We will utilize primarily a single reduction since this investigation uses BSE results from the heavy flavor sector.

The BSE provides an appealing starting point to describe hadrons as relativistic bound states of quarks. The BSE for a bound state may be written in momentum space in the form

$$G^{-1}(P,p)\psi(P,p) = \int \frac{1}{(2\pi)^4} V(P,p-p')\psi(P,p')d^4p', \tag{1}$$

where P is the four-momentum of the bound state, p is the relative four-momentum of the constituents. The BSE has three elements, the interaction kernel (V) and the propagator (G) which we provide as input, and the amplitude  $(\psi)$  obtained by solving the equation. We also solve for the energy, which is contained in the propagator.

Different approaches have been developed to make the four dimensional BSE more tractable and physically appealing. These include the Instantaneous Approximation (IA) and Quasi-Potential Equations (QPE) [5]. In the IA, the interaction kernel is taken to be independent of the relative energy. In QPE, the two particle propagator is modified in a way which keeps covariance and reduces the 4-dimensional BSE to a 3-dimensional equation. Of course, there is considerable freedom in carrying out this reduction.

Earlier, we have used two reductions of the QPE to study the meson spectrum [4]. These reductions correspond to different choices of the two particle propagator used to reduce the problem into three dimensions. We refer to these reductions as A and B. Reduction A corresponds to a spinor form of the Thompson equation [6] and reduction B corresponds to a new QPE introduced in Ref. [7]. These two covariant reductions are chosen because they are shown to give good fits to the meson spectrum. In both reductions, we assume the interaction kernel to consist of a one gluon exchange interaction,  $V_{OGE}$ , in the ladder approximation, and a phenomenological, long range scalar confinement potential,  $V_{CON}$  given in the form

$$V_{OGE} + V_{CON} = -\frac{4}{3}\alpha_s \frac{\gamma_\mu \otimes \gamma_\mu}{(p - p')^2} + \sigma \lim_{\mu \to 0} \frac{\partial^2}{\partial \mu^2} \frac{\mathbf{1} \otimes \mathbf{1}}{-(p - p')^2 + \mu^2}.$$
 (2)

Here,  $\alpha_s$  is the strong coupling, which is weighted by the meson color factor of  $\frac{4}{3}$ , and the string tension  $\sigma$  is the strength of the confining part of the interaction. We adopt a scalar Lorentz structure  $V_{CON}$  as discussed in [4]. In our formulation of BSE there are a total of seven parameters: four masses,  $m_u = m_d$ ,  $m_s$ ,  $m_c$ ,  $m_b$ ; the string tension  $\sigma$ , and two other parameters used to govern the running of the strong coupling constant. We varied these parameters to get the best fit for a list of known mesons as described in [4].

In our subsequent work [3] on the  $B_c$  meson, we evaluated the  $B_c$  spectrum without changing the parameter values mentioned above ( see Table 1 below ) and compared our results with those of Eichten and Quigg [8] and Gershtein at al. [9] using both the Martin potential and Buchmuller-Tye (BT) potential. The first row of Table 1 should be compared with the experimental result [1] of  $6.40 \pm 0.39$  (stat.)  $\pm 0.13$  (syst.) GeV/ $c^2$ . We have also evaluated the inclusive c-quark and  $\bar{b}$ -quark decay lifetimes [3] and obtained a  $B_c$  lifetime of 0.46-0.47 ps in good agreement with the experimental  $B_c$  lifetime of 0.46-0.46-0.16 (stat.)  $\pm 0.03$  (syst.) ps [1].

We now turn our attention to exclusive decays. The  $B_c$  exclusive semileptonic and nonleptonic decays have been discussed in the literature [9, 10, 11]. The effective Hamiltonian for the semileptonic decays has the standard current-current form, and is given by

$$H_W = \frac{G_F}{\sqrt{2}} V_{Qq} \bar{q} \gamma_{\mu} (1 - \gamma_5) Q \bar{\nu} \gamma^{\mu} (1 - \gamma_5) l.$$
 (3)

The leptonic current is completely known and the matrix element of the vector  $(V_{\mu})$  and the axial vector  $(A_{\mu})$  hadronic currents between the meson states are represented in terms of form factors which are defined by (considering the channel  $B_c \to B_s(B_s^*)$ )

$$\langle B_s(P')|V_{\mu}|B_c(P)\rangle = f_+(P+P')_{\mu} + f_-(P-P')_{\mu},$$

Table 1: Spectrum of  $B_c$  mesons in different channels (GeV/ $c^2$ ).

State	Our work	Our work	Eichten and Quigg	Gershtein et al.[9]	Gershtein et al.[9]
	Reduction A	Reduction B	Ref. [8]	Martin potential	BT potential
$1^{1}S_{0}$	6.356	6.380	6.264	6.253	6.246
$1^{3}S_{1}$	6.397	6.415	6.337	6.317	6.337
$1^{3}P_{0}$	6.673	6.692	6.700	6.683	6.700
$1^{3}P_{2}$	6.751	6.773	6.747	6.743	6.747
$1^{1}P_{1}$	6.752	6.777		6.729	6.736
$2^{1}S_{0}$	6.888	6.874	6.856	6.867	6.856
$2^{3}S_{1}$	6.910	6.891	6.899	6.902	6.899
$1^{3}D_{1}$	6.984	6.955	7.012	7.008	7.012

$$\langle B_s^*(P',\varepsilon)|V_{\mu}|B_c(P)\rangle = ig\epsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}(P+P')^{\alpha}(P-P')^{\beta},$$
  
$$\langle B_s^*(P',\varepsilon)|A_{\mu}|B_c(P)\rangle = f\varepsilon_{\mu}^* + (\varepsilon^*.P)[a_+(P+P')_{\mu} + a_-(P-P')_{\mu}]. \tag{4}$$

 $f_+, f_-, g, f, a_+$  and  $a_-$  are Lorentz invariant form factors which are scalar functions of the momentum transfer  $q^2 = (P - P')^2$  where P and P' are the four-momenta of the  $B_c$ ,  $B_s$   $(B_s^*)$  mesons respectively.

In our formalism, the mesons are taken as bound states of a quark and an anti-quark. We construct the meson states as [12]

$$|M(\mathbf{P}_{\mathbf{M}}, J, m_{J})\rangle = \sqrt{2M} \int d^{3}\mathbf{p} \langle Lm_{L}Sm_{S}|Jm_{J}\rangle \langle sm_{s}\bar{s}m_{\bar{s}}|Sm_{S}\rangle$$

$$\Phi_{Lm_{L}}(\mathbf{p})|\bar{q}(\frac{m_{\bar{q}}}{M_{q\bar{q}}}\mathbf{P}_{\mathbf{M}} - \mathbf{p}, m_{\bar{s}})\rangle|q(\frac{m_{q}}{M_{q\bar{q}}}\mathbf{P}_{\mathbf{M}} + \mathbf{p}, m_{s})\rangle, \qquad (5)$$

where the quark states are given by

$$|q(\mathbf{p}, m_s)\rangle = \sqrt{\frac{(E_q + m_q)}{2m_q}} \begin{pmatrix} \chi^{m_s} \\ \frac{\sigma \cdot \mathbf{p}}{(E_q + m_q)} \chi^{m_s} \end{pmatrix},$$

$$M_{q\bar{q}} = m_q + m_{\bar{q}},$$

$$E_q = \sqrt{m_q^2 + \mathbf{p}^2}.$$
(6)

In the above equations M is the meson mass. The meson and the constituent quark

states satisfy the normalization conditions.

$$\langle M(\mathbf{P'_M}, J', m_J') | M(\mathbf{P_M}, J, m_J) \rangle = 2E\delta^3(\mathbf{P'_M} - \mathbf{P_M})\delta_{J',J}\delta_{m'_{I},m_J}, \tag{7}$$

$$\langle q(\mathbf{p}', m_s')|q(\mathbf{p}, m_s)\rangle = \frac{E_q}{m_q} \delta^3(\mathbf{p}' - \mathbf{p})\delta_{m_s', m_s}.$$
 (8)

The wavefunctions  $\Phi_{Lm_L}$  appearing in Eq.(5) for the mesons are calculated by solving reductions of Bethe-Salpeter equation [4]. We have applied this formalism to evaluate the semileptonic form factors of the B to D and  $D^*$  mesons and showed that our results [13] are consistent with the heavy quark effective theory (HQET). We use wavefuctions from reduction B as we did in our previous work on B decays [13].

The values of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements we use in this paper are  $V_{ud} = 0.974$ ,  $V_{us} = 0.2196$ ,  $V_{ub} = 0.0033$ ,  $V_{cd} = 0.224$ ,  $V_{cs} = 0.974$ ,  $V_{cb} = 0.0395$  [14].

In Fig. 1 we show the semileptonic form factors for  $B_c \to B_s(B_s^*)$  and in Table 2 we show the exclusive semileptonic decay widths to different pseudoscalar and vector final states  $(B_c^+ \to P(V)e^+\nu)$ . We also compare our results with those of [11]. We notice in Fig. 1 that, although the semileptonic form factors are qualitatively similar to the  $B \to D(D^*)$  ones [13], flavor symmetry is absent in  $B_c$  decays as discussed in [15].

Table 2: Exclusive semileptonic  $B_c^+ \to P(V)e^+\nu$  decay widths in  $10^{-6}$  ev.

	Process	Decay width	Decay width
		This work	Chang and Chen [11]
	$B_c^+ \to \eta_c e^+ \nu$	11.1	14.2
$\bar{b}$ decay	$B_c^+ \to J/\psi e^+ \nu$	30.2	34.4
	$B_c^+ \to D^0 e^+ \nu$	0.049	0.094
	$B_c^+ \to D^{*0} e^+ \nu$	0.192	0.269
	$B_c^+ \to B_s^0 e^+ \nu$	14.3	26.6
c decay	$B_c^+ \to B_s^{*0} e^+ \nu$	50.4	44.0
	$B_c^+ \to B^0 e^+ \nu$	1.14	2.30
	$B_c^+ \to B^{*0} e^+ \nu$	3.53	3.32

For non-leptonic decays, the effective Hamiltonian (considering the  $B_c^+ \to B_s \pi^+$  channel) may be written as

$$H_W = \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^* [c_1(\mu) O_1 + c_2(\mu) O_2], \tag{9}$$

where

$$O_{1} = (\bar{u}_{i}d_{i})_{V-A}(\bar{s}_{j}c_{j})_{V-A},$$

$$O_{2} = (\bar{u}_{i}d_{j})_{V-A}(\bar{s}_{j}c_{i})_{V-A},$$
(10)

with (i, j = 1, 2, 3) denoting color indices and V - A referring to  $\gamma_{\mu}(1 - \gamma_5)$ .  $c_1(\mu)$  and  $c_2(\mu)$  are short distance Wilson coefficients computed at the scale  $\mu$ . By factorizing matrix elements of the four-quark operator contained in the effective Hamiltonian of Eq.(9), one can distinguish three classes of decays [16]. The first class ( class I ) contains those decays in which only a charged meson can be generated directly from a color-singlet current, as in  $B_c^+ \to B_s \pi^+$ . A second class of transitions ( class II ) consists of those decays in which the meson generated directly from the current is neutral, like the  $\pi^0$  meson in the decay  $B_c^+ \to B^+ \pi^0$ . Class I decay amplitudes are proportional to  $a_1$ , class II decay amplitudes are proportional to  $a_2$  where

$$a_1 = c_1(\mu) + \xi c_2(\mu),$$
  
 $a_2 = c_2(\mu) + \xi c_1(\mu),$  (11)

and  $\xi = 1/N_c$ , where  $N_c$  is the number of quark colors, and  $\mu$  is the scale at which factorization is assumed to be relevant. For the third class ( class III ) the  $a_1$  and  $a_2$  amplitudes interfere. Although the QCD factors  $a_1$ , and  $a_2$  have been calculated beyond the leading logarithmic approximation [17], we will follow the prevailing convention of theoretical predictions and express our results in terms of them. As an example the  $B_c^+ \to B_s \pi^+$  amplitude takes the form

$$A(B_c^+ \to B_s \pi^+) = \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^* a_1(\mu) < \pi^+ |(\bar{u}_i d_i)_{V-A}| 0 > < B_s |(\bar{s}_j c_j)_{V-A}| B_c > . \quad (12)$$

The matrix elements  $\langle B_s|(\bar{s}_jc_j)_{V-A}|B_c\rangle$  in Eq.(12) have already been evaluated in semileptonic decays of the  $B_c$  meson in terms of form factors, while the other matrix element  $(\langle \pi^+|(\bar{u}_id_i)_{V-A}|0\rangle)$  is related to the decay constant of the relevant meson. The weak decay constants  $f_P$  and  $f_V$  for pseudoscalar and vector mesons are defined by

$$\langle 0|J_{\mu}|P(p)\rangle = if_{P}p_{\mu},$$
  

$$\langle 0|J_{\mu}|V(p)\rangle = M_{V}f_{V}\varepsilon_{\mu},$$
(13)

where P and V are pseudoscalar and vector states, respectively, and  $J_{\mu} = V_{\mu} - A_{\mu}$  is the weak current ( $V_{\mu}$  and  $A_{\mu}$  are the vector and axial vector currents). The decay constants can be expressed in terms of the wavefunctions of the relevant mesons and are given by [18]

$$f_i = \sqrt{\frac{12}{M}} \int_0^\infty \frac{p^2 dp}{2\pi^3} \sqrt{\frac{(m_q + E_q)(m_{\bar{q}} + E_{\bar{q}})}{4E_q E_{\bar{q}}}} F_i(p), \tag{14}$$

$$F_P(p) = \left[1 - \frac{p^2}{(m_q + E_q)(m_{\bar{q}} + E_{\bar{q}})}\right] \psi_P(p), \tag{15}$$

$$F_V(p) = \left[1 - \frac{p^2}{3(m_q + E_q)(m_{\bar{q}} + E_{\bar{q}})}\right] \psi_V(p), \tag{16}$$

where  $\psi_{P(V)}$  are the momentum wavefunctions of the pseudoscalar (vector) mesons.

We have previously applied this formalism to evaluate the decay constants and the non-leptonic decays of the B mesons [19]. The values of the decay constants we use in this paper are  $f_{\pi}=0.130$  GeV,  $f_{\rho}=0.208$  GeV,  $f_{K}=0.159$  GeV,  $f_{K^*}=0.214$  GeV,  $f_{D}=0.209$  GeV,  $f_{D^*}=0.237$  GeV,  $f_{D_s}=0.213$  GeV,  $f_{D_s^*}=0.242$  GeV,  $f_{\eta_c}=0.400$  GeV,  $f_{J/\psi}=0.400$  GeV. These values are the available experimental ones [14]. Otherwise we use our values reported in [19]. These values of the decay constants are similar to those used by other authors [9, 10, 11].

In Table 3 we compare our results for the exclusive non-leptonic  $B_c \to PP, PV, VV$  decay widths of different channels where the  $\bar{b}$  quark decays with those of [11], while in Table 4, we make the same comparison for the case of c quark decays. At first glance, our decay widths in Table 3 are generally smaller than those of Ref. [11] by 20-40%. However, this is not a uniform trend as our  $B_c^+ \to D^+ \overline{D^{*0}}$  is 10% larger than that of Ref. [11]. If we furthermore compare total lifetimes for  $B_c$  we find that our lifetime (0.46 ps) is longer compared to Ref. [11] (0.40 ps) which is consistent with the dominant trends seen in the comparisons of the exclusive channels. Both theoretical lifetimes are well within current experimental uncertainties. Thus, experimental results for a set of exclusive channels could resolve between these two sets of theoretical predictions. Table 4 displays even greater range of differences between our model and that of Ref. [11].

In conclusion, we have systematically evaluated the decay widths of the exclusive semileptonic channels  $B_c \to P(V)e\nu$  and the exclusive two-body non-leptonic decays  $B_c \to PP$ , PV, and VV assuming that either c or  $\bar{b}$  quark inside the  $B_c$  meson is a spectator quark, and using our relativistic model [4]. In general, our predicted widths are smaller than those reported in Ref. [11] but there are exceptions to this trend. The variations between the theoretical predictions are wide enough so that experimental results should be able to discern

Table 3: Exclusive non-leptonic decay widths of the  $B_c$  meson in  $10^{-6}$  ev.  $\bar{b}$  quark decays with c quark spectator. The authors of Ref. [11] did not report the widths of some of the channels because it was thought, prior to the experimental discovery of the  $B_c$  meson, that these channels will be kinematically closed.

Class	Process	Decay width	Decay width
		This work	Chang and Chen [11]
	$B_c^+ \to \eta_c \pi^+$	$a_1^2 1.59$	$a_1^2 2.07$
	$B_c^+ \to \eta_c \rho^+$	$a_1^2 3.74$	$a_1^2 5.48$
	$B_c^+ \to J/\psi \pi^+$	$a_1^2 1.22$	$a_1^2 1.97$
	$B_c^+ \to J/\psi \rho^+$	$a_1^2 3.48$	$a_1^2 5.95$
I	$B_c^+ \to \eta_c K^+$	$a_1^2 0.119$	$a_1^2 0.161$
	$B_c^+ \to \eta_c K^{*+}$	$a_1^2 0.200$	$a_1^2 0.286$
	$B_c^+ \to J/\psi K^+$	$a_1^2 0.090$	$a_1^2 0.152$
	$B_c \to J/\psi K^{*+}$	$a_1^2 0.197$	$a_1^2 0.324$
	$B_c^+ \to D^+ \overline{D^0}$	$a_2^2 0.633$	$a_2^2 0.664$
	$B_c^+ \to D^+ \overline{D^{*0}}$	$a_2^2 0.762$	$a_2^2 0.695$
	$B_c^+ \to D^{*+} \overline{D^0}$	$a_2^2 0.289$	$a_2^2 0.653$
	$B_c^+ \to D^{*+} \overline{D^{*0}}$	$a_2^2 0.854$	$a_2^2 1.080$
II	$B_c^+ \to D_s^+ \overline{D^0}$	$a_2^2 0.0415$	$a_2^2 0.0340$
	$B_c^+ \to D_s^+ \overline{D^{*0}}$	$a_2^2 0.0495$	$a_2^2 0.0354$
	$B_c^+ \to D_s^{*+} \overline{D^0}$	$a_2^2 0.0201$	$a_2^2 0.0334$
	$B_c^+ \to D_s^{*+} \overline{D^{*0}}$	$a_2^2 0.0597$	$a_2^2 0.0564$
	$B_c^+ \to \eta_c D_s^+$	$(a_1 2.16 + a_2 2.57)^2$	$(a_1 1.13 + a_2 1.98)^2$
	$B_c^+ \to \eta_c D_s^{*+}$	$(a_1 \cdot 2.03 + a_2 \cdot 2.16)^2$	$(a_1 1.04 + a_2 1.90)^2$
III	$B_c^+ \to J/\psi D_s^+$	$(a_1 \cdot 1.62 + a_2 \cdot 1.72)^2$	$(a_1 \cdot 1.02 + a_2 \cdot 1.95)^2$
	$B_c^+ \to J/\psi D_s^{*+}$	$(a_1 3.13 + a_2 3.67)^2$	
	$B_c^+ \to \eta_c D^+$	$(a_10.485 + a_20.528)^2$	$(a_10.193 + a_20.440)^2$
	$B_c^+ \to \eta_c D^{*+}$	$(a_10.466 + a_20.452)^2$	$(a_10.181 + a_20.430)^2$
	$B_c^+ \to J/\psi D^+$	$(a_10.372 + a_20.338)^2$	$(a_10.177 + a_20.442)^2$
	$B_c^+ \to J/\psi D^{*+}$	$(a_10.686 + a_20.732)^2$	

between the models.

Table 4: Exclusive non-leptonic decay widths of the  $B_c$  meson in  $10^{-6}$  ev. c quark decays with  $\bar{b}$  quark spectator.

Class	Process	Decay width	Decay width
		This work	Chang and Chen [11]
	$B_c^+ \to B_s^0 \pi^+$	$a_1^2 15.8$	$a_1^258.4$
	$B_c^+ \to B_s^0 \rho^+$	$a_1^2 39.2$	$a_1^2 44.8$
	$B_c^+ \to B_s^{*0} \pi^+$	$a_1^2 12.5$	$a_1^2 51.6$
	$B_c^+ \to B_s^{*0} \rho^+$	$a_1^2 171.$	$a_1^2 150.$
I	$B_c^+ \to B_s^0 K^+$	$a_1^2 1.70$	$a_1^2 4.20$
	$B_c^+ \to B_s^{*0} K^+$	$a_1^2 1.34$	$a_1^2 2.96$
	$B_c^+ \to B_s^0 K^{*+}$	$a_1^2 1.06$	
	$B_c^+ \to B_s^{*0} K^{*+}$	$a_1^2 11.6$	
	$B_c^+ \to B^0 \pi^+$	$a_1^2 1.03$	$a_1^2 3.30$
	$B_c^+ \to B^0 \rho^+$	$a_1^2 2.81$	$a_1^2 5.97$
	$B_c^+ \to B^{*0} \pi^+$	$a_1^2 0.77$	$a_1^2 2.90$
	$B_c^+ \to B^{*0} \rho^+$	$a_1^29.01$	$a_1^2 11.9$
	$B_c^+ \to B^0 K^+$	$a_1^2 0.105$	$a_1^2 0.255$
	$B_c^+ \to B^0 K^{*+}$	$a_1^2 0.125$	$a_1^2 0.180$
	$B_c^+ \to B^{*0} K^+$	$a_1^2 0.064$	$a_1^2 0.195$
	$B_c^+ \to B^{*0} K^{*+}$	$a_1^2 0.665$	$a_1^2 0.374$
	$B_c^+ \to B^+ \overline{K^0}$	$a_2^2 39.1$	$a_2^2 96.5$
	$B_c^+ \to B^+ \overline{K^{*0}}$	$a_2^2 46.8$	$a_2^268.2$
	$B_c^+ \to B^{*+} \overline{K^0}$	$a_2^2 24.0$	$a_2^273.3$
	$B_c^+ \to B^{*+} \overline{K^{*0}}$	$a_2^2 247$	$a_2^2 141$
II	$B_c^+ \to B^+ \pi^0$	$a_2^2 0.51$	$a_2^2 1.65$
	$B_c^+ \to B^+ \rho^0$	$a_2^2 1.40$	$a_2^2 2.98$
	$B_c^+ \to B^{*+} \pi^0$	$a_2^2 0.38$	$a_2^2 1.45$
	$B_c^+ \to B^{*+} \rho^0$	$a_2^2 4.50$	$a_2^2 5.96$

We note that the dominant decays are those when the b quark inside the  $B_c$  meson behaves as a spectator quark and a vector meson is produced in the final state. In fact,  $B_c^+ \to B_s^{*0} e^+ \nu$  is the dominant decay among all the semileptonic channels (see Table 2) and  $B_c \to B_s^{*0} \rho^+$  becomes the dominant among all the two-body non-leptonic decays (see

Table 4). Although these decays are suppressed by phase space they are CKM favored.

Finally we point out that the ratio

$$\frac{\Gamma(B_c^+ \to V \rho^+)}{d\Gamma(B_c^+ \to V e^+ \nu)/dt|_{t=m_\rho^2}} = 6\pi^2 |V_{ud}|^2 a_1^2 f_\rho^2, \tag{17}$$

with  $V = B_s^{*0}$ ,  $J/\psi$  will be a good experimental test for the numerical value of the coefficient  $a_1$  of QCD [16].

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### Figure Caption

Fig. 1 The semileptonic form factors for  $B_c \to B_s(B_s^*)$ .

